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INCORPORATING REDUNDANCY CONSIDERATIONS INTO STOCKAGE MODELS

INVENTORY
RESEARCH
OFFICE

ALAN J. KAPLAN

JANUARY 1987

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This paper is concerned with evaluating the impact on weapon system availability of component and assembly redundancy. The evaluation must be efficient, and it must be possible to integrate the evaluation into multi-echelon stockage models whose objective is to find the least cost mix of stockage consistent with the availability goals for weapon (or other type) systems.

The mathematics to be discussed here provides a rigorous solution to the evaluation problem when there is only a single supply echelon; there can be upper echelon repair, but not supply, unless the supply is from a "perfect" supplier, always in stock. For the more general multi-echelon case, approximate approaches are presented.

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INCORPORATING REDUNDANCY CONSIDERATIONS INTO STOCKAGE MODELS

1. INTRODUCTION

1.1 Objective.

This paper is concerned with evaluating the impact on weapon system availability of component and assembly redundancy. The evaluation must be efficient, and it must be possible to integrate the evaluation into multi-echelon stockage models whose objective is to find the least cost mix of stockage consistent with the availability goals for weapon (or other type) systems.

The structural representation of a system to be supported is illustrated in Figure 1, wherein each component of the same type is assigned the same number. The type "5" components are in parallel, so that if either component is up, the system is up. Components 7-8-9 constitute an assembly. If exactly one component 7 and one component 8 as well as both component 9s are up, the system item is up if and only if the operating component 7 and operating component 8 belong to the same assembly. Component 1 is a redundant component in a redundant assembly, the assembly consisting of components 1 and 2. (Component 1 is shown more for its pedagogical value than its realism).

A multi-echelon supply system is illustrated in Figure 2 wherein only one base is shown. When a component is removed from the system, it is the responsibility of site supply to provide a replacement. This replacement is unlikely to be the component just removed, unless the site happens to have a zero stockage level for that component and the component can be repaired at the site.

The mathematics to be discussed here provides a rigorous solution to the evaluation problem when there is only a single supply echelon; there can be upper echelon repair, but not supply, unless the supply is from a "perfect" supplier, always in stock. For the more general multi-echelon case, approximate approaches are presented.

1.2 Assumptions.

a. Time to component failures is exponential.

b. All components of the same type have the same failure rate. They are subject to failure so long as they are installed in the system, this is referred to as "warm standby," and are also subject to failure during installation.

c. Component failures are independent of each other.

d. Component and assembly redundancy is of the "k of n" type; that is, k of n components or assemblies in parallel must function for the system to be up.

e. Components within assemblies are replaced as they fail, the failure being identified by built in test equipment. There is no cannibalization of components and components are replaced in order of failure.

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COMPONENT STRUCTURE
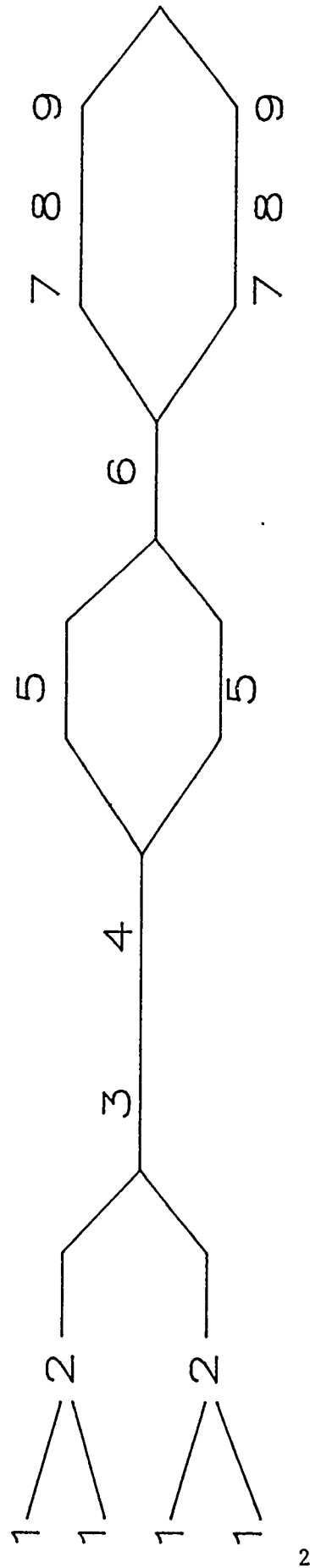
ASSEMBLY STRUCTURE
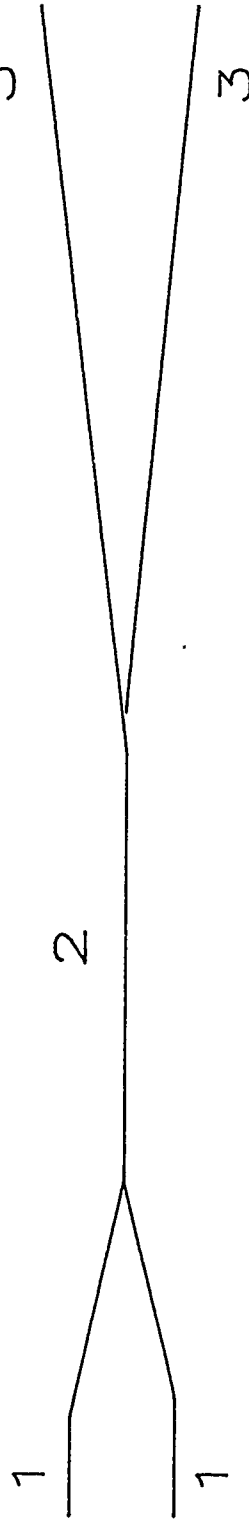
COMPONENT 1 AND 5 ARE REDUNDANT
ASSEMBLY 1 CONSISTS OF COMPONENTS 1 AND 2
ASSEMBLY 2 CONSISTS OF COMPONENTS 3-4-5-6
ASSEMBLY 3 CONSISTS OF COMPONENTS 7-8-9
ASSEMBLIES 1 AND 3 ARE REDUNDANT
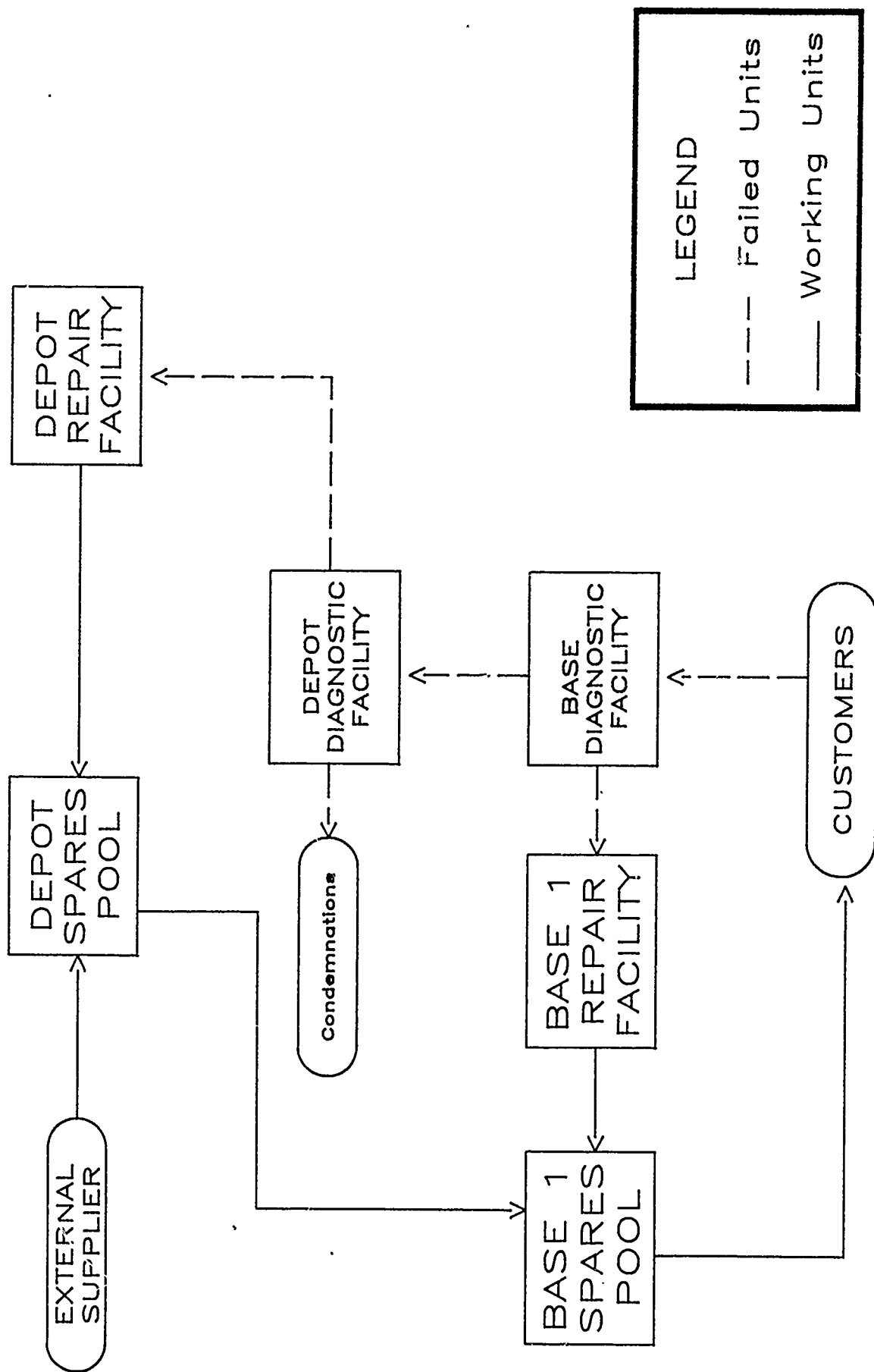
FIGURE 1: SYSTEM STRUCTURE

FIGURE 2: FLOW OF UNITS IN A TWO-ECHELON SUPPLY SYSTEM

f. The disposition of a failed component, whether it can be repaired at all, and at which echelon, is determined in accordance with a series of Bernoulli trials.

g. The times to procure new components are independent and identically distributed for all components of the same type; repair times are also i.i.d, but may differ by echelon; installation times are deterministic, but may depend on the component type, and ship times are also deterministic.

h. All supply points, except perhaps the supplier at the top echelon, use a one for one ordering policy, i.e., they order each time they get a failure they cannot repair.

i. Suppliers fill backorders on a first come, first served basis.

j. The performance objective of interest is the steady state expected value of the number of systems operational.

The set of assumptions described are consistent with the SESAME multi-echelon stockage model used by the U. S. Army [14], as well as with other implemented stockage models which are descendants of the METRIC model [11]. However, for convenience, such models have generally replaced assumption b, above, with the infinite source assumption that the failure rate experienced by a supplier does not depend on the number of components installed, and so is unaffected by backorders. At the site level, for parallel redundancy of two components, the finite source effect was found to be important [Kaplan, 1979].

1.3 Related Work. In the 1960's, Amster and Morra [1] working at Bell Laboratories developed a single echelon inventory model for redundant components, assuming each supplier supported a single system. A steady state difference approach was used, so that consistent with this approach, repair, resupply, and installation times were all assumed to be exponential.

Gross, Miller, and Soland [4] extend the steady state difference approach to model a multi-echelon system with finite repair capacity and finite source demand on the supplier. Their work could be adapted to redundant components and assemblies. However, the original article treated only one bottom echelon site, and computational efficiency is a major problem with added sites. Gross and Miller report some success in modelling transient behavior in the more general case [5] and additional research is underway.

2. COMPONENT REDUNDANCY

In this section, and the next on assembly redundancy, we will assume that inventory theory can provide us with the steady state distribution of backorders at the site supply point. We will then examine that assumption more closely.

Let

M: number of systems supported by a site.

C: number of components (of a given type) per system.

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I: installation time.

f: failure rate per component

B(x;s,m): the probability that a random variable with binomial
distribution, probability of success s, and m trials,
realizes the value x.

b_t: number of backorders at time t at a site.

z_t: total non-operating components (of a given type) at time t
at a site.

Let t be a random point in time. If we look at the components in a
system at time (t-I), then they can be in one of three states: operating;
being installed; missing because of a site backorder. Those components which
are operating or being installed at time (t-I) will be operating at time t,
unless they fail in the interim. If they do fail, there will not be
sufficient time to complete installation of a replacement. Likewise, those
components missing at time t-I cannot be obtained and operating by time t.

In accordance with the assumptions numbered a-c, if there are m
components either operating or being installed at time t-I, the probability
that x will fail is B(x; s,m) where $s = 1-e^{-fI}$. Then, if there is only one
system supported by a site,

$$Pr(z_t=z) = \sum_{b=0}^{z} Pr(b_{t-I}=b) \ B(z-b; \ 1-e^{-fI}, \ C-b) \qquad (2.1)$$

If there is k of n redundancy for a component type, the probability it is not
a cause for the system to be down is the probability $z_t \leq n - k$.

When there are M systems supported by a site, the hypergeometric
distribution describes the probability that exactly b of x total missing
components are missing from a particular system i:

$$P \ (b_{ti}=b) = \sum_{x=b}^{(M-1)C+b} Pr(b_t=x) \ \frac{\binom{C}{b} \binom{(M-1)C}{x-b}}{\binom{MC}{x}} \qquad (2.2)$$

Use of the hypergeometric is justified by assumptions a-c and e which make all
assignments of backorders to systems equally likely. The upper bound on the
summation in (2.2) arises because (M-1)(C) is the number of components, and
therefore the maximum number of backorders, on systems other than "i".

Once the distribution for the number of missing components on a
system chosen at random is found, the multi-system case reduces to the single
system case. $Pr(b_{t-I,i}=b)$ is substituted for $Pr(b_{t-I}=b)$ in equation (2.1).

There is an obvious analogy between the approach underlying equation
(2.1), and the approach used to calculate the distribution of on hand assets

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at time t in steady state single echelon inventory models (cf Hadley and Whitin). Such models look at total assets including due-in at time $t-L$, where L is procurement lead time. Demand in the lead time plays a role in inventory theory comparable to failure in the installation time here.

Note that although components _fail_ independently (assumption c), stockage introduces correlation between the operational status of components of the same type; for example, in Figure 1, the operational status of the two component "5's" is correlated. We will discuss why in more detail later. This correlation is implicit in the backorder distribution used in calculating the z_t in equation (2.1). Because of correlation, neither component nor assembly redundancy can be modelled by the simpler approaches possible when the status of all components is independent (cf Barlow and Proschan).

3. ASSEMBLY REDUNDANCY

Let

   M: number of systems supported.

   A: number of assemblies of a given type per system.

   z_i: number of component type i not operating at a random point in time (time subscript is suppressed).

   Y: number of component types in the assembly; that is, for the example of Figure 1, for the assembly made up of components 7-8-9, Y = 3.

Renumber the component types in the assembly beginning with 1, so that if Y = 3, the component types are numbered 1, 2, 3. Define:

   a_i: number of assemblies at a random point in time which would not be operational even if all components of types i + 1 to Y were functioning.

a_Y is the number of assemblies not operating at a random point in time. For k of n assembly redundancy, the assembly is not a cause of the system being down if $a_Y \leq n - k$. The distribution of a_Y is calculated recursively: Thus, $a_{m+1} = j$ if $a_m = w$, $w \leq j$, and component type m + 1 accounts for exactly j - w additional assemblies to be down. When there is one system supported per site, the equations are:

$$\Pr(a_1=j) = \Pr(z_1=j) \tag{3.1}$$

$$\Pr(a_{m+1}=j) = \sum_{w=0}^{j} \Pr(a_m=w) \sum_{k=j-w}^{j} \Pr(z_{m+1}=k) \cdot \text{Allocation } (k,j-w)$$

where Allocation (k,j-w) is the probability that the k non-operating components of type m + 1 account for exactly j - w additional assemblies to be down.

$$\text{Allocation } (k,j-w) = \frac{\binom{A-w}{j-w}\binom{w}{k-(j-w)}}{\binom{A}{k}}$$

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The denominator is the total number of ways of assigning k non-operating components of type m + 1 to the A assemblies; e.g. if k = 3 and A = 5, one way would be that component type (M + 1) was inoperable on assemblies 1, 4, and 5. The numerator is the total number of ways of assigning the k components while satisfying the criterion that exactly j - w of the k are assigned to the (A-w) operating assemblies.

The generalization of (3.1) to multiple systems per site is handled by first calculating a_Y without distinguishing which system the assembly is on.

a. The z_i used in (3.1) are defined as the number of components of type i inoperable on all systems. There are (M)(A) components of type i per site.[1] Hence, (M)(A) is substituted for C in equation (2.1) to get these z_i.

b. In the calculation of Allocation (k,j-w), (M)(A) is substituted for A.

c. a_{Yi} is defined as the number of assemblies not operating on system i. The distribution of a_{Yi}, conditional on the value of a_Y, is hypergeometric:

$$Pr(a_{Yi}=j) = \sum_{w=j}^{(M-1)A+j} Pr(a_Y=w) \; \frac{\binom{A}{j}\binom{(M-1)A}{w-j}}{\binom{MA}{w}} \qquad (3.2)$$

4. BACKORDER DISTRIBUTION, SINGLE ECHELON

Let

S: stockage level.

v: number of components in repair or due-in from procurement (of a particular type, at a random point in time).

b: number of components backordered.

N: number of components at the site subject to failure when backorders are 0.

f: failure rate per component.

r: resupply rate per component (reciprocal of mean resupply time to the site).

g(v): probability mass function for v.

In the current context, resupply refers to all sources of due-in to the site supply point, whether from site repair, higher echelon repair, or

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[1] The thorough reader will note this is not true for component 1 in Figure 1, but the methodology discussed is readily generalized to that case also.

procurement. Based on assumptions f and g, the resupply distribution is a mixture of the distributions for resupply time from each source, and is i.i.d.

The stockage level is, by definition, the sum of on hand plus due-in minus backorders. It is a constant. When the system is fielded all S units are on hand; thereafter each demand debits on hand or increases backorders, while also increasing due-in, in accordance with assumption h.

By definition, then,

$$b = \max (0, S - v) \tag{4.1}$$

The distribution of b is computed from the distribution of v. The number in resupply, v, corresponds to a finite state, birth - death process with state dependent arrival and service rates, exponential interarrival time and general service distribution. Demands are births, and completion of resupply are deaths. The state is the number in resupply. Sherbrooke, [10] drawing on earlier proofs by others, showed that the steady state distribution of v is the same for general service as it is for exponential service distributions.

In particular then, (cf Gross and Harris),

$$g(v) = g(0) \prod_{i=1}^{v} \frac{f_{i-1}}{r_i} \tag{4.2}$$

where r_i is the resupply rate and f_i the arrival rate when $v = i$:

$$r_i = (i)(r)$$

$$f_i = N \cdot f \qquad\qquad i \leq S$$

$$f_i = [N - (i-S)]f \qquad\qquad i > S$$

f_i, $i > S$, shows the effects of the (i-S) backorders on the arrival rate. Note that f_{i-1}, not f_i, appears in the numerator of (4.2). g(0) is a normalizing constant so that the probabilities sum to 1.

Equation (4.2), after substituting for r_i and f_{i-1}, and letting $\lambda = f/r$, becomes:

$$g(v) = g(0) \frac{\lambda^v N^v}{v!} \qquad\qquad v \leq S$$

$$g(v) = g(0) \frac{\lambda^v N^S}{v!} \frac{N!}{(N-v+S)!} \qquad\qquad S + 1 \leq v \leq N + S \tag{4.3}$$

In the computer implementation the g(v) are computed without reference to g(0), and are never revised; instead, g(0) is factored directly into (2.1), outside the summation. Computational efficiency is also obtained by computing and storing factorials once per computer run.

5. SPECIAL CASE: NO STOCKAGE

As mentioned earlier, stockage introduces correlation into the operating status of different components of the same type. Intuitively, if a component is backordered, this means the supplier is out of stock, which makes it more likely a second component, failing independently, will also be backordered. When there is no stockage, the supplier is always out of stock, so that the status of one component has no implications for others.

Thus for the case of zero stockage, there are two approaches to computing the probability of z non-operating components: the approach of Section 2, and the classical approach using the binomial probability distribution (cf Barlow and Proschan). We will not bore the reader here with all the algebra necessary to show the two approaches are equivalent, the algebra is in the Appendix, but two aspects are of at least academic interest and are discussed here.

5.1 Probability a Component is Not Operating.

Given assumptions a-c and g of Section 1, we derive the probability that a component chosen at random is not operating.

Let

        f: failure rate

        r: resupply rate

        F: mean time between failures (equals $1/f$)

        R: mean time to resupply (equals $1/r$)

        I: time to install, if component does not fail during installation.

        p: probability of a failure during installation.

        E: expected time to failure during installation, given that a failure occurs.

        T: mean total down time after a failure.

        D: expected fraction of time a component is down (not operating).

Then

$$D = \frac{T}{T+F} \tag{5.1}$$

$$T = R + (1-p) I + p (E+T) \tag{5.2}$$

$$E = \frac{1}{p} \int_0^I tfe^{-ft}$$

$$= \frac{f}{p} \frac{1}{f^2} [1 - e^{-fI}(1 + fI)]$$

Using, $p = 1-e^{-fI}$, E reduces to

$$E = \frac{1}{f} - \frac{I(1-p)}{p} \tag{5.3}$$

Substituting (5.3) into (5.2),

$$T = (\frac{1}{1-p}) [R + (1-p) I + p (\frac{1}{f} - I \frac{(1-p)}{p})] \tag{5.4}$$

$$= \frac{1}{1-p} [R + \frac{p}{f}]$$

Substituting (5.4) into (5.1), and defining $\lambda = f/r$,

$$D = \frac{R+p/f}{R+p/f+(1-p)F} = \frac{fR+p}{fR+1} = \frac{\lambda+p}{\lambda+1} \tag{5.5}$$

5.2 Calculation of the Normalizing Constant.

In Section 4 we noted that the number in resupply corresponded to the state of a finite source birth-death process. For such processes it is usually not possible to compute the normalizing constant, $g(0)$ in our notation, without first computing the entire distributions (cf Gross and Harris). When stockage is 0, however, it turns out $g(0)$ can be obtained very easily, as we will now show.

When $S = 0$ equation (4.3) reduces to

$$g(v) = g(0) \lambda^v \binom{N}{v} \qquad\qquad 0 \le v \le N \tag{5.6}$$

Hence,

$$\frac{1}{g(0)} = \sum_{v=0}^{N} \binom{N}{v} \lambda^v \tag{5.7}$$

The following identify is in Riordon [9]:

$$\sum_{k=0}^{n} \binom{n}{k} x^{n-k} = (1+x)^n$$

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Rewriting (5.7) and then applying the identify with $x = 1/\lambda$:

$$\frac{1}{g(0)} = \frac{N}{\lambda} \sum_{v=0}^{N} \binom{N}{v} \lambda^{v-N} = (\frac{1}{\lambda})^{-N} \sum_{v=0}^{N} \binom{N}{v} (\frac{1}{\lambda})^{N-v}$$

$$\quad (5.8)$$

$$= (\frac{1}{\lambda})^{-N} (1 + \frac{1}{\lambda})^{N} = (\lambda+1)^{N}$$

Hence:

$$g(0) = (\lambda + 1)^{-N} \quad\quad\quad (5.9)$$

6. DISTRIBUTION OF BACKORDERS, MULTI-ECHELON

In the multi-echelon case, resupply times to the bottom echelon sites are not independently distributed, and as a result the number in resupply at the bottom echelon, the v of Section 4, are not Poisson.

Ignoring for a moment the implications of finite source, the required site backorder distribution may be determined exactly [8], or by a very good and efficient computational approximation proposed independently by Slay [12] and Graves [2]. This approximation, called Varimetric by Slay, is utilized by the U. S. Army [14] in its standard multi-echelon stockage model. The approximation involves computing the mean and variance of v exactly, and using the Negative Binomial with this mean and variance as the (approximate) distribution of v.

Now, how can we adjust Varimetric for finite source? Our concern focused on the base (bottom echelon) where the adjustment would intuitively seem to be most critical.

Now equation (4.3) may be rewritten as

$$g(v) = k\ p(v; N\cdot\lambda) \quad\quad\quad v \leq S$$

$$\quad (6.1)$$

$$g(v) = k\ p(v; N\cdot\lambda)\ a(N,v,S) \quad\quad S + 1 \leq v \leq N + S$$

where k is a normalizing constant, $p(v; N\cdot\lambda)$ is the Poisson probability mass function with parameter $(N\cdot\lambda)$, and $a(N,v,s)$ is a finite source adjustment function.

We posited that use of the same function, $a(N,v,S)$, would work reasonably well in adjusting the distribution of v in the multi-echelon case wherein the infinite source distribution of v is approximated as Negative Binomial. Specifically, if $n(v;-,-)$ is the Negative Binomial probability mass function, with parameters appropriate to the infinite source case,

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$$g(v) \doteq k \cdot n(v; -, -) \qquad\qquad v \leq S$$

$$g(v) \doteq k \cdot n(v; -,) \cdot a(N, v, s) \qquad S + 1 \leq v \leq N + S \qquad (6.2)$$

$$a(N, v, S) = \frac{N! N^S}{(N-v+S)! \ N^V}$$

Note, of course, that the normalizing constant in (6.2) does not have the same value as it would in (6.1).

Zmurkewycz tested the approximation in a two-echelon simulation with a depot (upper echelon supplier) and four bases. The depot supply process and the base supply process for one base, the base of interest, were simulated in detail. The other three bases collectively generated demands on the depot, but otherwise did not figure in the simulation.

In this simulation, the depot experienced demand as if demands were from an infinite source, while the base of interest generated demand reflecting the finite source implication. Specifically, there were two components at each base, so that when the mean time between failure (MTBF) was $1/d$, demands on the depot were generated at a rate of $8/d$, 8 being the total number of components (4 bases, 2 components per base). However, when a demand was generated, it was assigned to the base of interest with probability $c/8$, when c was the number of operating components there at the time the demand was generated. (c takes on the values 0, 1, or 2).

Thus, any discrepancies between analytic projections using (6.2) and simulation results would be due in small measure to use of Varimetric, and otherwise to use of the $a(N, v, S)$ adjustment functions.

Simulated results were compared to analytic projections for varying MTBFs and varying stock levels. The base stock level was set at 0 or 1, while the depot stock level was set at the mean number in resupply to the depot, rounded, with a minimum value of 1. If the depot has 0 stock, the multi-echelon case reduces to a single echelon, and the analytic projections are exact; this fact was used in testing the simulation computer code. There were six days to get a component from depot to base and 30 days to resupply the depot. Base repair was not simulated, since base repair reduces the impact of the problem being investigated; with 100 percent base repair, the analytical projections are exact.

Table 1 shows the results of the simulation work. The analytic projections of mean and variance of backorders, as developed from equation (6.2), are presented ("E-3" means 10^{-3} as in FORTRAN; 3 significant digits are shown in the table). Contrasted to these projections are the simulation results, and analytic projections made without any adjustment for finite source. The precision of the simulation reported relates to mean backorders only. It represents the estimated standard deviation of mean backorders output by the simulation as a percent of the mean. Average backorders were computed for each year of the simulation after a 2-years warmup, and the sample standard deviation of these yearly observations was used to develop the

TABLE 1. EVALUATION OF APPROXIMATION

MTBF (DAYS)	ORG TARGET STOCK	DSU TARGET STOCK	ANALYTIC RESULTS				SIMULATION RESULTS		
			FINITE SOURCE MEAN	FINITE SOURCE VARIANCE	INFINITE SOURCE MEAN	INFINITE SOURCE VARIANCE	MEAN	VARIANCE	PRECISION
40	0	6	.394 E0	.330 E0	+37.3%	+90.8%	+2.0%	+1.4%	0.2%
	1	6	.108 E0	.118 E0	+35.8%	+80.5%	-0.4%	-0.2%	0.6%
80	0	3	.259 E0	.234 E0	+22.8%	.52.0%	-0.2%	-0.1%	0.3%
	1	3	.481 E-1	.527 E-1	+21.6%	+46.4%	+0.8%	+0.6%	0.6%
160	0	2	.131 E0	.126 E0	+11.1%	+23.7%	-1.0%	-1.2%	0.3%
	1	2	.129 E-1	.138 E-1	+10.3%	+21.6%	-0.2%	-0.5%	0.9%
320	0	1	.869 E-1	.848 E-1	+7.1%	+14.8%	-1.2%	-1.1%	0.3%
	1	1	.572 E-2	.600 E-2	+6.5%	+13.4%	-0.4%	-0.8%	1.0%
480	0	1	.496 E-1	.490 E-1	+4.2%	+8.6%	-1.3%	-1.3%	0.3%
	1	1	.198 E-2	.204 E-2	+3.9%	+8.0%	-2.6%	-2.7%	1.3%
640	0	1	.334 E-1	.332 E-1	+2.9%	+5.8%	-1.4%	-1.5%	0.3%
	1	1	.920 E-3	.942 E-3	+2.7%	+5.4%	-1.9%	-1.8%	1.6%

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estimate of precision. The number of years the simulation was run was proportional to the MTBF; it was as high as 250,000 years for an MTBF of 640 days.

Summarizing Table 1, the contrast between (approximate) finite source and infinite source projections is most pronounced for the lower MTBF, and more pronounced for the variance than the mean. The approximation worked well for the lower MTBFs and about equally well for mean and variance; the approximation did not improve, and tended to become worse, for the higher MTBFs where the total correction necessary was not great. The approximation tended to "under correct".

It should be noted that MTBFs less than 360 days are rare for the repair parts we deal with. Also, it is only for redundant components that the number of systems down depends on the variance as well as the mean of the backorder distribution.

In addition to the results reported in Table 1, the simulation was run for an MTBF of 420 days and 10 components at each base. Results were consistent with those in Table 1 with the simulation mean and variance differing from analytical by +0.3% and +0.4% when base stock was 0, and -2.0% and -1.9% when base stock was 1.

7. SPECIAL CASE: NO BASE STOCKAGE

No attempt was made in Section 6 to adjust for the finite source implications at the depot level. Fortunately, there is one case where a good adjustment can be made, and this is precisely the case wherein the depot supply performance has the most direct impact on the base: when base stock is zero. Base stock of 0 is a common output of the optimization process for reliable redundant components.

We have noted previously that when depot stock is zero, the multi-echelon reduces to the single echelon case; this is almost true when depot stock is positive and base stock is zero. The first thing we observe is that when base stock is zero, each depot backorder equates directly into a component missing from a system. Our next observation is that when base stock is zero, the only distinction from a modelling viewpoint between ship time from depot to base and installation time is that by assumption b a component may fail during installation, but not during shipment.

The approximation is to assume that components may fail during shipment. The algorithm is:

a. Determine depot backorders using the results of Section 4 with "N" being the number of components at all sites, and "r" referring to the resupply rate from outside supplier and/or depot repair to depot supply.

b. Determine total non-operating components at a site by using Section 3 with "I" being set to the sum of installation time plus ship time from the depot, "M" being the number of systems supported by the depot, "b_t" referring to backorders at the depot.

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8. CONCLUSION

For some systems redundancy is important, and this report demonstrates that it is quite possible to incorporate it into a multi-echelon stockage model. The approach taken fully utilizes inventory theory implemented for non-redundant components.

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APPENDIX

EQUIVALENCE OF APPROACHES

This appendix shows that for the case of zero stockage, the approach of Section 2 reduces to a simple Binomial. For case of exposition, we assume one system per site. Specifically then, we show the RHS of (2.1), with C equal to N, the number of components per site, reduces to $B(z;D,N)$ where $B(z;D,N)$ is the Binomial probability mass function with parameters D and N, and D is the expected fraction of time a component is not operating. We repeat from Section 5 two equations we will need (equations 5.5 and 5.6).

$$D = \frac{\lambda+p}{\lambda+1} \tag{A.1}$$

$$g(v) = g(0) \; \lambda^v \binom{N}{v} \qquad\qquad 0 \le v \le N \tag{A.2}$$

$g(v)$ as defined in A.2 is the probability mass function for the number of components in resupply when the component is not stocked. When the component is not stocked, number of components in resupply equals the number of backorders, so that equation (2.1) may be written:

$$Pr(z_t=z) = \sum_{v=0}^{z} g(v) \; B(z-v;1-e^{-fI},N-v) \tag{A.3}$$

Using A.2 and letting p represent $1-\exp(-fI)$,

$$Pr(z_t=z) = \sum_{v=0}^{z} g(0) \; \lambda^v \; \binom{N}{v} \binom{n-v}{z-v} \; p^{z-v}(1-p)^{N-z} \tag{A.4}$$

$$= g(0) \; \frac{N!}{(N-z)!} \; p^z(1-p)^{N-z} \sum_{v=0}^{z} \frac{1}{v!} \; \frac{1}{(z-v)!} \lambda^v p^{-v}$$

Multiplying by $1/z!$ outside the summation, and $z!$ inside the summation,

$$Pr(z_t=z) = g(0) \binom{N}{z} \; p^z \; (1-p)^{N-z} \sum_{v=0}^{z} \binom{z}{v} \left(\frac{\lambda}{p}\right)^v \tag{A.5}$$

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